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Research Article

Multiple Consequences Related to Atmospheric Turbulence Induced by the Climate Change in the Heatwaves Emergence and in the Cooling Effect of Aerosols

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Abstract

When dealing with the general problem of turbulence there are several theoretical and practical related problems: the generation (origin) of fluid fluctuations (real eddies and mathematical vorticity), the turbulent transfer of kinetic energy, heat and mass, drag resistance, clean-air fluctuations, hurricanes and tornadoes, atmospheric circulation and plumes, and other natural or human-induced phenomena. We are tempted by the intent to formulate a unified approach, where turbulence is the general feature of these problems. We attempt here to draw some connections between the theoretical turbulence modeling and the experimental results interpreted using such models and the reality of large-scale natural events strongly related to anthropogenic climate changes, such as heatwaves and the cooling effect of aerosols. In fact we believe that more sophisticated practical results could be drawn from connecting theoretical turbulence studies to natural real phenomena, especially those under the influence of climate change. The mathematical modeling aimed at increasing predictability did not produce yet a fundamental breakthrough in the understanding of turbulence. In dealing with real turbulent flows we constantly rely on phenomenological approaches. To date, the large-scale spatio-temporal characteristics of turbulence has yet to be fully understood, due to the lack of sufficient in situ detection instruments in the atmosphere. As such, there is much room for improvement in turbulence-related parameterizations in global weather and climate prediction models. Short presentations of the heatwaves and cooling effect of aerosols are considered from the point of view that the study of weather data and the use of statistical modeling should be coupled with the fundamental studies on the fluid dynamics features of turbulence which play the primary role in the atmospheric circulation and thus in weather and climate changes.

Keywords

Turbulence, Randomness, Vorticity, Heat Transfer, Mass Transfer, Atmosphere Circulation, Heatwaves, Arosols

1. Introduction

When dealing with the general problem of turbulence there are several theoretical and practical related problems: the generation (origin) of fluid fluctuations (real eddies and mathematical vorticity), the turbulent transfer of kinetic energy, heat and mass, drag resistance, clean-air fluctuations, hurricanes and tornadoes, atmospheric and oceanic plumes, and other natural or human-induced phenomena. We are tempted by the intent to formulate a unified approach, where

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turbulence is the general feature of these problems. Turbulence generates imprecision. The discrepancy between what exists and what can effectively be achieved in climate science and the necessity to bridge these two sets of concepts, constitutes in itself a source of imprecision. Very rarely it happens that a single kind of imprecision emerges alone. The different imprecisions tend to accumulate and give birth to a process of escalation of imprecision which effects are quite unpredictable. One clear example is the famous "butterfly effect" in dynamic weather systems. We aspire to quantify the imprecision or at least to discover solid qualitative patterns in the presence of it. Yet, the tension between formalization (rigor) and explanation (sense) is an organic source of imprecision. Climate science is a science of qualitative approximations, of probable or fuzzy rather than exact results. This fact is shown in the emergence and dynamics of turbulence movements in climate phenomena. They are increasingly not just a characteristic of natural fluid flows but also of climate change. Sometimes we encounter unexplained "self-breeding" turbulence in the study of real and theoretical vorticity. Turbulence is a main factor and permanent feature of atmospheric circulation dynamics, including jet streams. But we simply don't have a single, well established mathematical model of turbulence. The nature (origin) of turbulence is the great problem of fluid dynamics, thus also of climate science. Turbulence is not just one problem but it seems to be a set of problems and, for the time being, different approaches are needed.

An intensification of turbulent phenomena could have important consequences for human activities (weather heatwaves and floods- forecasting, waterworks, restoring natural areas, aviation, urban management, etc.). The real-world implications of the increase in flow resistance due to ubiquitous presence of turbulence are huge. A large fraction of the world's energy consumption is devoted to compensating for turbulent energy loss. Nevertheless, the detailed understanding and prediction from first principles still elude turbulence theory. The important feature is the quality of approximations represented mainly by the value of the order of magnitude of the error, the time necessary to get the approximation and the cost of the process of approximation (algorithms, computer power, energy consumed). We should also include the sources of error related to confirmatory significance testing [1]. In climate science specification testing is clearly important in order to achieve sufficient confirmation of the degree of approximation through mathematical modeling since it's impossible to have precise values in the forecasting. We can but should not use in climate science "serendipity as a governing structure", as Jé Wilson describes the unpredictability of literary prowess [2], although the unpredictability of changes related or triggered by human activities is so obvious. Turbulence is the main factor of interrupting the continuity in space and time of many fluid flows. Our knowledge of the atmospheric circulation and jet streams is not yet a well-established corpus of fluid dynamics science itself. In the last 30 years, the advent of very powerful scientific computation, combined with new capabilities in data acquisition and analysis, promised to radically improve the study of fluid turbulence. Yet, the fulfillment of this promise is still work in progress. Indeed, as Heisenberg firmly noted: "We have to remember that what we observe is not nature in itself but nature exposed to our method of questioning" [3]. Here Heisenberg referred not only to the quantic field but to classical physics in general.

Many important scientific observations arise when we explore the moment of change at very small scales. A key question in real-world situations is whether the necessary assumptions of homogeneity and isotropy are satisfied at small scales, thus justifying application of a general framework for those smaller scales. Typical turbulence encountered in the real world often obeys neither condition at large scales. Thus, it can be important to understand if there is a moment of disruption at very small scales within the atmospheric circulation or jetstreams. We attempt here to draw some connections between the theoretical turbulence modeling and the experimental results interpreted using such models and the reality of large-scale natural events strongly related to anthropogenic climate changes, such as heatwaves and the cooling effect of aerosols. In fact we believe that more sophisticated practical results could be drawn from connecting theoretical turbulence studies to natural real phenomena, especially those under the influence of climate change.

2. The Turbulence Problem: Phenomenology vs. Theoretical Approaches

Turbulent fluid flows, ubiquitous in nature and technology, are in fact strong and chaotic fluctuations in pressure and flow velocity across a wide range of interacting scales in space and time. Natural events are showing structure at many length scales. The most important property of turbulence is its ability to produce mixing, thereby transporting scalar quantities such as heat or aerosols both along and across surfaces of constant density, in the atmosphere or in the ocean. Associated with this fact is that turbulence is a strong interaction phenomenon, and highly dissipative. A complete understanding of turbulence requires knowledge on the kinetic energy dissipation rate, diffusion coefficient, inner and outer scales, heating, spectrum, seasonal variation, height and geography dependency, and generation mechanism. Thus, the study of turbulence is about understanding the mechanisms of turbulent energy, vorticity, and mass transfer between scales and between points in space. In these mechanisms, large velocity differences (not the velocity itself) resulting from shear forces applied to the fluid (or from intrinsic fluid instability) produce strong fluid turbulence, a state of spatial and temporal fluctuations that can be described by the Navier-Stokes equations. Turbulent interactions are highly non-linear, leading to mathematical intractability of the governing equations. This means that even when the behavior and characteristics of the phenomenon in the fluid flow are entirely deterministic (i.e. not random), the equations still give rise to chaotic behavior. In spite of intense research efforts, gathering information from experiments, observations and from computer simulations, associated with theoretical models explaining the turbulence features, our present understanding of the turbulence phenomena remains incomplete, often relying on phenomenological approaches. The flow is generally unpredictable in deterministic terms. Starting with Lorenz's celebrated paper [4], it became more and more important to understand if and how chaotic random-like behavior can be simulated with purely deterministic equations. Turbulent flows possess a range of scales. The largest scales are permanently influenced by the initial setting of the physical system in which the flow is taking place. Ample evidence indicates that the large scales of turbulent flow are not so chaotic and may display highly organized coherent structures. The problem then is to compute the universal statistical properties that all turbulent flows share despite their different large-scale driving mechanisms or their particular flow geometries. Besides, the dependency of velocity statistics at various temporal scales on large scale forcing and boundary conditions constitutes a problem of universality. An essential concept in the phenomenology of turbulence is that of small-scale universality, postulated by Kolmogorov [5], which forms the backbone of turbulence theories and models. The physical picture described using Richardson-Kolmogorov fundamental model is that the kinetic energy k injected at the largest scales L of the flow at an average rate ε, generates large-scale fluctuations. The injected energy cascades down to smaller scales via non-linear inertial, energy conserving processes until it reaches a scale of order l, where viscous dissipation becomes dominant and the kinetic energy is converted into heat. In other words, the intermediate spatial scales r, in the interval $l \ll r \ll L$, define an inertial range in which large-scale forcing and viscous forces have negligible effects. Energy cascading down through spatial scales is a central feature of modeling fluid turbulence. Many studies are devoted to improving the ability of the k-ε eddy viscosity model to predict complex turbulent flows. The standard paradigm is that whereas the large scales are non-universal, reflecting the circumstances which generated them, the non-universality is decreasing as the fluctuations decline towards the small-scales, displaying an increasing separation between the large and small scales. Small-scale dynamics are strongly non-linear even in low-Reynolds-number flows. The non-linear term in the Navier-Stokes equation produces essentially an eddy viscosity, i.e. the effect of very small scales of turbulence is just to change the viscosity into an eddy viscosity. This mathematical non-linearity is driven by large-scales when the solution is determined using a superposition of linear combinations of the results in homogeneous flows and the forcing term. This scale separation is thought to increase with the flow Reynolds number, so a proper test of universality has been thought to require very high Reynolds numbers. "If one resolves small scales accurately, one observes, even at low Reynolds numbers, universal scaling of velocity gradients that manifest primarily at small scales" [6]. Here we consider not the scale-size of average velocity itself, but the variance on distances of the size of L, because this variance is describing the velocity of turbulent movement. In large fluid bodies there is little connection between the motion at well separated points in the fluid, and energy propagates slowly, with the speed of the fluid motion. "In the interior of strongly stratified fluids, such as the atmosphere, turbulence can occur only in isolated patches. Those patches can arise as a result of the superposition of motions from many sources and on many scales" [7]. The distribution of a passive scalar concentration carried by a turbulent flow is very intermittent. "In other words" - explains Robert Ecke - "there is a much larger probability (compared with what one would expect for a random, or Gaussian, distribution) of finding local concentrations that differ greatly from the mean value. Kolmogorov's law is a statement of conservation of energy from scale to scale in the inertial regime of homogeneous isotropic turbulence" [8]. There exists a threshold Reynolds number above which Gaussian-like fluctuations acutely show intermittent characteristics of fully developed flows [9]. The *International Collaboration for Turbulence Research*, which studied Lagrangian structure functions, concluded that "A fundamental open question is connected to intermittency, i.e. the observed strong deviations from Gaussian statistics, becoming larger and larger when considering fluctuations at smaller and smaller scales" [10].

The dynamic behavior of a fluid system is characterized by a transition from an ordered state to a chaotic state. Essentially, such systems are chaotic because they display a strong sensitivity property of the initial conditions, i.e. the evolution of the system depends closely on the adopted/real initial condition. Several mechanisms leading to chaos (simply put, an unpredictable evolution) "depend on the trajectory in the parametrization plan" [11]. Some experiments show that the system, "crossing successively various possible states…mixed with temporary organized ones…doesn't stop in a stable condition". The scope of the theory of dissipative dynamic systems is to reconcile deterministic and stochastic structures. However, when many modes are effectively interacting, the theory's contribution is reduced.

A crucial development in turbulent studies on the origin of turbulence is the introduction of the *attractor* concept. Ruelle and Takens [12] demonstrated that hydrodynamic turbulence can be, in certain situations, related to the so-called *deterministic chaos* of dynamic systems with few degrees of freedom. The deterministic feature is related here to the fact that an underlying "order" is present, i.e. a hidden deterministic feature which can be analyzed in a phase space. Under this concept a random mixing in time is governed by quantitative laws, meaning that pure hazard is not anymore the cause of the mixing. The deterministic character of such chaos is real. Chaotic dynamical systems display predictable structures in phase space where the chaos is mathematically explained by the theory of strange attractors. That's why the probabilistic dimension in the attractor space is in fact an information dimension because the system, losing its initial condition reference, becomes a generator of information [13]. Such systems have one or even several attractors. But these results are not based directly on the hydrodynamical equations because of insurmountable calculations needed. In such cases we assume that the flow is described by partial differential equations and we compute them using linearized dynamical equations. Moreover, attractors refer only to temporal chaos. Their capacity for predicting chaotic behavior, of limited practical importance, remains certainly important in the endeavor of explaining the origin of turbulence.

A striking feature of chaotic dissipative systems such as turbulence is the spontaneous emergence of coherent structures, within the space of chaotic interactions [14]. In turbulent flows, the lasting presence of coherent "worms" of intense vorticity - "eddies''naturally associated with the idea of a vortex - is a specific example. Understanding the mechanisms by which intense vorticity structures are generated and controlled is essential to assess their relevance to turbulence dynamics. The causal relations that drive this mechanism are not well understood due to its nonlinear and non-local nature. As an example, "two of the most intriguing aspects of intense vortices are their disparity of scales and the scaling of their circulation" [14]. There are numerous attempts of using eddy viscosity formulation in various real turbulent cases of newtonian and non-newtonian fluids (as presented, for example, in the studies [15, 16]). They have radii of the order of Kolmogorov units, but lengths that reach up to the inertial – or even integral – scales, and their circulation vary with the root-mean-square of the velocity fluctuations and the Kolmogorov length scale. "Observations suggest that large and small scales are involved in their dynamics, but it remains unclear which scales control their formation and evolution. There is evidence that points in the direction of a top-down mechanism controlled by large scales" [14]. Thus, it can be imagined a separation of the turbulent flow in two regions: an active one, driven by the Kolmogorov cascade model, and a weak turbulent background, independent of the same model. In that sense, two or more chaotic systems evolving simultaneously to a common pattern are synchronized if they are driven by similar large-scale dynamics.

Results indicate the role of large-scale dynamics in the formation of intense vorticity structures and rule out the possibility that vorticity emerges primarily due to interactions within the dissipative range. Their evolution is connected to scales at the end of the inertial range, growing or coalescing into filament structures. It is important, both theoretically and practically, to understand whether they constitute a relevant feature of turbulent flows or they are just the byproduct of other underlying turbulent processes, with reduced influence on the overall dynamics. So, "in the case of results which do not involve specifically the physical properties of the flow, no adaptation of them is possible when the equilibrium conditions (physical and chemical) are changed" [17].

All our mathematical modeling aimed at increasing predictability did not produce yet a fundamental breakthrough. Indeed, "predictability estimates based on turbulent phenomenology do not describe in principle the coherent structures mentioned above. Real dynamics and the theoretical chaotic dynamic systems do have common features only in few turbulent flows and for not long time-lengths. So far, the presentation has remained purely phenomenological" [9].

3. Turbulent Flows Generating Unstable Patterns

For a fluid flow to be considered stable it must be stable with respect to every possible disturbance. A fluid flow is unstable if any external perturbations will generate in the state of the system a disturbance growing in amplitude in such a way that the system never returns to the initial condition. Transitions from laminar to turbulent flows show a variety of patterns. It can trigger chaotic behavior only in time or both in time and space. A very weak perturbation may result in a complete disruption of the flow pattern. However, the statistical time averages based on a vast amount of data can be stable and in that frame predictable. But, as Benoit Mandelbrot demonstrated [18], the very small frequency of extraordinary, unpredictable events "beats" the massiveness of average data. Predicting weather for a longer than 48 hours period of time is not possible, while climate change, computed as a statistical average, is predictable. Chaotic behavior of dynamical systems doesn't mean complete lack of predictability. Small but finite perturbations which start growing exponentially eventually saturate at some not very high level. Controlled experimental turbulent settings have displayed coherent predictable structures. It must be outlined that stability analysis doesn't provide a reliable indication about the resilience (robustness) of the system under the impact of sequences of small shocks or disturbances or simultaneous small shocks that together make up a big perturbation.

In atmospheric flows, the turbulence of marked particles (Lagrangian turbulence) is in general unpredictable when the large-scale flow is not steady. It could be that, like in some experimental types of turbulence, the large-scale flow becomes more coherent, while at the small-scale the behavior is chaotic [9]. The main difficulty of Lagrangian investigations, following particle trajectories, stems from the necessity to resolve the wide range of time scales driving different particle behaviors. Moreover, a chaotic process controls the formation of coherent structures. In the atmosphere, there are several mechanisms triggering instability which are acting on very different scales. The question then is if it is possible to segregate the coexisting turbulent flows (they are separated by a gap) in order to refine the computational models in view of increasing the predictability of weather, for example. The resolution of the spectral gap (in terms of wave numbers) that exist at scales of approximately 100 km (that being the smallest resolvable details) should increase predictability over days. However, the intimate unpredictability related to the energy spectrum, can generate violent meteorological events. In this case the statistical mean significance is lost.

The models commonly used, such as those based on Richardson - Kolmogorov energy cascade of eddies theory and the Monin-Obukhov Similarity Theory (MOST), do not offer satisfactory predictions in cases of extremely unstable conditions dominated by high turbulence intensity generated in turbulent mixing zones in which many different eddies interact [19, 20]. MOST is applied to measured time series data from meteorological masts at the proposed wind farm location to understand the effects of atmospheric stability and to obtain the characteristic values (Table 1), [20].

Table 1. *The Monin-Obukhov Length (MOL) classification of atmospheric stability.*

Condition	Monin-Obukhov Length [m]
Extremely Unstable	
Unstable	
Neutral	|L| > 500
Stable	
Extremely Stable	

It is interesting to note that this classification displays a fuzziness pattern. Fuzziness and randomness are distinct forms of uncertainty and imprecision. When we deal with a random experiment, we are often interested in functions of the experimental outcomes rather than the outcomes themselves. The general fuzzy framework encompasses both statistical uncertainty, which falls within the field of probability theory, and non statistical uncertainty which relates to the concept of a fuzzy set and possibility theory [21]. The descriptive definition is suitable for purposes of analyzing and proving most of the properties of fuzzy random variables. Given a random experiment, and a probability space modeling this experiment, traditionally it is assumed that the experimental performance is accomplished under randomness, and the quantification process associated with the experimental outcomes is real-valued. That’s why the fuzziness method could be envisaged in the treatment of natural phenomena data dominated by the turbulence induced randomness. For example, turbulence based fuzzy time series forecasting is effective, especially, when the available data indicate a high degree of instability [22].

In a turbulent regime of flow it could be that an experiment developed in the same physical conditions but at different moments leads to different results. That’s why Landau's relation in connection with Kolmogorov and Obukhov ideas is valid only statistically [23]. Landau noticed that any model of the local structure of turbulence should take into account the essentially accidental and random character of the mechanism of transfer of energy from the coarser vortices (eddies) to the finer. Turbulent flows “enjoy a very large number of degrees of freedom …and the set of exact initial conditions data, which would determine the values of such a very large number of magnitudes, is to such a degree inimaginable that posing the problem in that manner is senseless from the physical point of view” [23]. Landau's equations concerning the local instability properties of turbulence (stationary critical situations) or the HopsOscillatory instability are certainly instructive for the understanding of the transition to (weak) turbulence, but we have to consider situations of higher criticality. Beyond the instability threshold (in the emergence of turbulence) the number of interaction modes is so large that radically new phenomena occur and develop. Manneville, referring to the specificity of the transition scenarios, explain clearly that: “ in the continuum space of developing instabilities, the number of modes of interaction is primarily and much more related to the geometric characteristics of - *forme factors* or *aspect relationships* - than to the number of the distinct physical processes” [24]. An important feature is that this kind of disorder (instabilities) is simultaneously spatial and temporal. From this follows an important question concerning the spatial organization of turbulence: an observed gap in time also reflects spatial development? In fact we have to consider, in real phenomena, that the spatial and temporal chaos are intimately related, i.e. the structure shows a dynamical disorder. All fully developed turbulent flows display such structures. As a matter of fact we have to deal not only with the chaos itself but also with the characterization of the emergence of chaos, especially in experimental studies, after a certain instability threshold is crossed.

4. Measuring and Simulating Turbulent Flow in the Atmospheric Circulation

In the layers of the atmosphere circulation (the jet stream pattern included) turbulence might be nearly homogenous in a region much larger than the so-called “continuous central core region” [25]. The turbulent quantities decrease toward the outer region due to the intermittent character of turbulent flow, namely “in the boundary regions between the mixing zones and the undisturbed free stream outside it” [25], pg. 285. It is useful to measure the degree of intermittence of turbulence, playing a decisive role in some movements in the atmosphere, as a ratio between the time during which turbulence occurs and the total time. However, the problem of measurements of turbulent parameters is formidable in real atmospheric flows. “There are some among us who consider turbulence and its measurement to be a black art. There are others who criticize because they perceive a lack of proof of the validity of the measurements that are reported; and there are some of us who must recognize that some of our earlier results are indeed suspect. However, all is not as bad as it might sometimes seem " as Bruce Hicks put it [26]. Understanding the mechanisms of turbulence, dispersion and heat transfer is becoming increasingly important. For example, simulating numerically the flows over a genuine urban region in order to provide the dynamic large scale inlet boundary conditions, requires a continuous specification of appropriate inlet turbulence. Aggregating data from near-surface meteorological networks for initiating dispersion models is examined in [27]. The problem raised is: "How dense must such a network be to yield useful information and what measurements should be reported?" Although many *in situ* wind and mass concentration measurements are available their utility is limited because specifically turbulence data which should be used in *k-ε* eddy viscosity model, for instance, are more often than not insufficient and non-homogeneous. Curtailing this fact requires an adequate understanding of the processes that control velocity and turbulence within the surface roughness layer. Indeed, it has to be stressed that in this case as in many other similar ones the measurements of wind velocity are not measurements of turbulent fluctuations, which remain difficult to detect due to their small size and high variability. Blind extrapolation of existing analysis to other situations has to be avoided. To date, the large-scale spatio-temporal characteristics of turbulence has yet to be fully understood, due to the lack of sufficient *in situ* detection instruments in the atmosphere. As such, there is much room for improvement in turbulence-related parameterizations in global weather and climate prediction models (see the analysis for non-convectively driven turbulence in the free atmosphere, [28, 29], and also, the analysis of experimental data using an algorithm based on the Kolmogorov–Obukhov law, [30]).

Vorticity is another related feature. The turbulence generated in fluid flows that cross obstacles and form a drag zone is well studied. The Richardson-Kolmogorov energy cascade theory explains the eddies spectrum of turbulence and the decay of vorticity down to the viscous inertial subrange where friction dissipates the kinetic energy. We suggest the question whether the vorticity in the free atmosphere is related to the eddies spectrum? If so, is it a continuum one or does it include some qualitative change. This question might be essential in order to understand the chaotic changes in the atmospheric circulation triggered by turbulence.

Using the work of Rizzo and Rapisarda [31], Beck, Cohen and Rizzo analyzed the statistical properties of turbulent wind velocity fluctuations at Florence Airport [32]. The data were recorded by two head anemometers A and B on two poles 10 m high a distance 900 m apart at a sampling frequency of 5 minutes. Components of spatial wind velocity differences at the two anemometers A and B as well as of temporal wind velocity differences at A were investigated. Analyzing these data, the authors distinguished two well separated time scales. On the one hand, the temporal velocity difference fluctuates on a rather short time scale. On the other hand, they looked at a measure of the average activity of the wind bursts in a given longer time interval (1 hour in their analysis). Adopting a local variance parameter of the macroscopic turbulent fluctuations, they found that, for a given non-equilibrium system, the probability density of this parameter is ultimately determined by the underlying spatio-temporal dynamics of the system under consideration. Their conclusion is of interest to our above mentioned suggestion: "So far there is no theory of turbulence, but following Kolmogorov [5], the mechanism of the turbulent motion of the fluid is critically determined by the transfer mechanism of the energy dissipation between neighboring cells and between different spatial scales in the flow". The significant small-scale motions contributing most to the dissipation of energy must be intermittently distributed through the flow and Kolmogorov assumed that the rate of dissipation was constant wherever there was significant small-scale motion. However, in a work devoted to Kolmogorov's work [33], the authors conclude that "for the majority of phenomena connected with small-scale turbulent motions these corrections can be neglected in the statistical description of the turbulence (when there is no local sampling of special 'events')". Also we should have in mind that in the fundamental approach of Kolmogorov there is an implication that the turbulence changes slowly on the natural time-scale of the small eddies. In the situations that need local sampling, the authors [33] raise the question if the universality of small-scale concept is valid in all particular turbulent patterns. Furthermore, is it possible for a turbulent flow to consist of an ensemble of elements with no internal structure? Complete answers to these questions are not yet available.

We advance here another question: in which way the basic randomness of turbulence is related to the fact that in many processes, random individual events can, in aggregate, lead to highly deterministic outcomes as indicated by Michel Talagrand? His fundamental question is: "What can be said about the maximum value of a random variable over a certain range of time? How can we guarantee that, with probability close to one, this maximum will not exceed a given threshold?" [34], preface. If measurements of random variables are independent, then "the totals become very predictable, even if each individual event is impossible to predict… Even though something has so much randomness, the randomness cancels itself out" and "What initially seemed like a horrible mess is actually organized" [34]. This phenomenon, known as *concentration of measure*, occurs in much more complicated random processes, too. Giorgio Parisi, founder of the theory of complex systems, acknowledged Talagrand's result as explaining his mathematically unresolved complex pattern of spin glass: "My belief was that (the spin glass) was a problem so difficult it could not be proved…It was a wonderful (Talagrand's) proof and completely changed the situation, because this was a starting point for a much deeper understanding of the theory" [35]. Theoretically, computed solutions of the Navier-Stokes equations show the importance of isolated eddies with complex internal structure of intense vorticity. Flow visualization and computer simulation show that most of the eddies in turbulent flows are formed during the interaction between eddies, which are often large vortices. Using Talagrand observation we need to calculate the effects of turbulence on various processes in the atmosphere that may (or may not) interfere significantly with the free circulation flow and find if, when and how the *concentration of measure* pattern exists in the atmospheric turbulence and jet streams. Also, in relation to the study of jet streams, the broad-spectrum view of meteorology adopted by Herbert Riehl stressed the importance of the role occupied by the potential vorticity field in their structure and formation [36]. He observed that a large entrainment of mass into the air current is taking place, and this is typical of many situations where a jet stream intensifies downstream. We think that the vorticity set in the atmospheric circulation could be a result of interferences en masse, possibly related to dynamical processes that are acting in the energy transfer from large to small scales and in particular to the eddies conservative energy transfer.

5. The Emergence of Heatwaves

Global warming from increasing greenhouse gasses has and will continue to increase heat wave hazards. The emergence of heatwaves (HW) are manifested as local intraseasonal phenomena and they result from large-and small-scale processes that interact in complex ways and at a wide range of temporal scales. The fragmentary understanding of the physical drivers contributing to HWs is aggravated by a fundamental lack of understanding of dynamical aspects. Among the influential factors is the atmospheric circulation, typically considered a fast driver. While frequency and intensity of HWs and heavy rainfall events are expected to increase in a warming world due to thermodynamic arguments, the exact location and duration of these events are more uncertain and largely controlled by the atmospheric circulation, especially at mid-latitudes [37]. The authors indicate that “Large-scale weather systems typically move eastward, but when the jetstream strongly meanders this transport can come to a halt”. Global greenhouse gas (GHG) concentrations and regional (land-use/land-cover, aerosols) anthropogenic forcings are the dominant factors of long-term trends in the frequency, duration and intensity of HWs. Moreover, increasing levels of GHG can also cause changes in atmospheric circulation. All in all, there is limited dynamical understanding of the factors determining the onset of HWs in the atmospheric systems. The current global climate models (GCM) are a result of continuous refinements added by climate scientists, based on larger and better data. Yet, the uncertainty of climate projections at regional scales is still large. One, if not the major problem resides precisely in the far from complete understanding of the fundamental processes. We are permanently confronted with the need to model different uncertainties using various concepts and methods. All these uncertainties continue compromising the gathering of useful information without which no strategy is reliable. “Overall, the physical drivers of HWs are not well understood, due partly if not mainly to difficulties in the quantification of their interactions and responses to climate change” [38]. It is well documented that midlatitude HWs, like the Western European in 2003, the Russian in 2010, the China’s in 2022 [39] are due to rare and complex dynamical anomalies leading to a stationary pattern of the jet stream. Studying extreme heat waves then amounts to studying the nonlinear and turbulent dynamics of the atmosphere. Two key dynamical variables to be studied are the temperature and pressure fields. Our climate system is not just a static thermodynamic system, it is a fluid dynamical system. The physical drivers and anticyclone blocking events are still to be properly modeled and the theoretical basis for their future change is unknown. It’s this fact that makes us believe that the study of weather data and the use of statistical modeling should be coupled with the more fundamental studies on the fluid dynamic features of turbulence which play the primary role in the atmospheric circulation and thus in weather and climate changes. “If a finite visible structure cannot be transformed into an infinite intelligible structure, we don't have an act of creation. It's just craftsmanship” elegantly stated the Romanian mathematician Solomon Marcus [40].

6. The Cooling Effect of the Aerosol Droplets

The large-scale motions and the overall dynamics of the flow determine the magnitude of the motions in any given flow. The instability of the fluid motion may lead to a large amplification of the random forces which are an effective force felt at smaller length scales that results from the turbulent, but deterministic, motion at larger scales L.

The study of atmospheric turbulent flows with small heavy particles suspended - like aerosol droplets - usually raise two distinctive questions [41]:

1) the particles are homogeneously distributed in space or do they form clusters? and

2) what is the average collision velocity between the particles?

In other words, are the particles brought together by the turbulent flow velocity or the particles detach from the flow and thrown in the general current move closer towards each other? This question is in contrast to the usual static-local measurements not associated with fluid dynamics models of transportation.

Understanding the spatial distribution of finite-size massive impurities, such as droplets, dust, or bubbles suspended in incompressible flows is a crucial issue in cloud physics. Such particles possess inertia, and generally are distributed in a strongly inhomogeneous manner. The common understanding of this long known but remarkable phenomenon of preferential concentrations relies on the idea that, in a turbulent flow, vortices act as centrifuges ejecting particles heavier than the fluid and entrapping lighter ones. Particle dynamics in the inertial range can be directly related to the structure of the pressure field (and thus of acceleration). “Characterizing the distribution of acceleration is thus crucial to understand particle clusters” [42]. The authors investigated the statistical properties of velocity gradients along trajectories of fluid tracers, heavy and light particles, and found that around the dissipative time lags, these particles behave strongly differently, due to the effect of being expelled/concentrated out/in vortex filaments. The dynamics is strongly influenced by the geometry of the underlying flow.

Let’s imagine a stable circulation generated by temperature and/or pressure gradients. The flow at the microscale level hitting the aerosol droplets generates very small eddies which form tiny drag zones. The question we suggest is if these eddies, present in such a big number in the free flow, might interact and form a large pattern of vorticity sufficient to “derail” the stable atmospheric circulation. And maybe these eddies are entities that cannot be discerned from one another. In such situations, eddies don’t have well-defined identity conditions and cannot be accounted for by standard means. Of course, if we have a collection of indistinguishable eddies and some of them have some property, then all of them will have the property. What we need is a representation given by mathematical structures applied to real entities. But it must be underlined that we can directly apply mathematics (and logic) to reality only in some restrictive cases; the general account requires representation employing mathematical structures, most currently sets. It seems that sets are not, for now, the preferred tool in studying turbulence.

The above mentioned pattern of non-equilibrium doesn’t fit the Richardson-Kolmogorov theory since the generation mechanism of such a pattern probably includes a critical threshold. Research on the detection of turbulence states indicated the existence of non-equilibrium scaling in laboratory experiments as well as in the atmosphere [43].

Using high-resolution wind velocity data collected during helicopter flights in the eastern North Atlantic inside the 10 km by 10 km square adjacent to the Graciosa island [44], the authors show that non-equilibrium states are also present in the stratocumulus-topped boundary layers, which indicates the presence of rapidly changing external conditions. They also indicate deviations from Kolmogorov's −5/3 law. These classical relations do not describe non-equilibrium turbulence states which appear to be present inside the stratocumulus cloud. The question of the existence of some correlation between the vorticity pattern generated by microscale eddies and the non-equilibrium scaling turbulence is wide open. Nothing can be proved until we are able to measure the kinetic energy of the microscale eddies.

Strong events at once could reinforce or cancel out each other’s influence on the global climate in complex ways. It might be good news in one place, it might be bad news somewhere else. As an example, the aerosol droplets have acute complex effects. How much radiation is reflected by sulfur dioxide aerosols varies according to the size of the droplets, their height in the atmosphere, whether it is night or day, what season it is and several other factors. These complexities mean there is still a great deal of uncertainty about the magnitude of the overall cooling from pollutants such as SO_2. But if aerosol cooling is larger than generally assumed, the planet will warm more rapidly than predicted as soon as aerosol levels fall. Now we think firmly that the uncertainties are certainly reduced if less CO_2 is emitted. But, if aerosol cooling is on the higher side, there will be more warming because fossil fuels tend more and more to be phased out, and as a result aerosol pollution falls. Aerosols can influence the Earth’s climate in two ways. When the sky is clear (devoid of clouds), aerosols can reflect incoming sunlight back to outer space – the direct effect. This blocks part of the energy that would have reached the surface, thus having a cool effect on the climate. The second (indirect) effect is that more aerosols may also enable clouds to last longer by suppressing rainfall. Despite many years of active research, aerosols are still the least certain of all known climate forcings. We think that the studies of particle movements in turbulent flows, as summarized in a quick manner in this article, have to be seriously considered along with the local-static measurements and statistical analyses. Aerosols are not so much counteracting global warming as constantly postponing it. Now we think firmly that the climate change uncertainties are certainly reduced if less CO_2 is emitted. But if aerosol cooling is larger than generally assumed, the planet will warm more rapidly than predicted as soon as aerosol levels fall. Financing one policy strongly - like eliminating greenhouse gas as much as possible - doesn’t necessarily bring the predicted outcome. H. Murakami [45] suggests that substantial changes in large-scale circulations, caused by the changes in anthropogenic aerosols, led to the changes in global spatial distribution of tropical cyclones. In fact, this study underscores the importance of multiple consequences induced by anthropogenic activity.

An experimental study [46] determined the time dependence of the aerosol droplets' mean radius upon initiation of flow in an oscillating grid generated turbulence chamber. The authors investigated the rate of aerosol coalescence in a well characterized turbulent flow, using measurements of the evolution of the mean aerosol radius upon initiation of the turbulent flow, along with measurements of the initial number density of the drops. The time dependence of the aerosol droplets mean radius upon initiation of flow in an oscillating grid generated turbulence chamber is determined using a phase-Doppler method. Together with a measurement of the aerosol number density from a light attenuation probe, the observed rate of change of the aerosol droplets mean radius can be related to the rate constant for the coalescence of two droplets. In another study [47], measurements in the atmosphere show that the coupling between the various mechanisms driving coalescence is not additive and, on the other hand, the overall collision efficiency is significantly lower than one. Turbulent fluctuations of atmospheric aerosol were studied experimentally at tropospheric altitudes of 2–6 km above the ground in Israel. Various data indicate that the spectrum of passive scalar fluctuations is not universal for the entire range of scales (and frequencies). The authors concluded that many turbulence analyses should be re-evaluated for applications above the boundary layer.

Aerosol–cloud–precipitation interactions represent one of the major uncertainties in weather and climate prediction [48]. Current atmospheric models cannot resolve the microphysical processes and thus rely on parameterizations to represent those interactions. Studies show that model results of the location and intensity of precipitation are sensitive to microphysics schemes. The authors investigate the relative importance of turbulence, CCN hygroscopicity, and aerosols (size and number concentration) on the DSD (droplet size distribution) broadening in cumulus clouds. Turbulence sustains the formation of large droplets by effectively accelerating the collisions of small droplets. The DSD broadening through turbulent collisions is significant. The results show that "turbulence and cloud condensation nuclei (CCN) hygroscopicity are key to the efficient formation of large droplets". It is also suggested that "a turbulence-dependent relative-dispersion parameter should be considered".

7. Cooling Surface Sea Temperature in the Eastern Tropical Pacific

Many climate models fail to simulate the Eastern Tropical Pacific (ETP) sea surface temperature (SST) wedge-shaped cooling phenomenon [49]. This study shows that "an increase followed by a decrease in anthropogenic sulfate aerosol emissions can produce a multidecadal cooling pattern in the ETP". In another research [50], the authors have examined the fast and slow components of tropical Pacific SST responses to anthropogenic sulfate aerosol emissions. They observed that the equatorial Pacific displays opposing signs of fast (surface) and slow (subsurface) components, and both components are insensitive to exact aerosol distributions outside the equatorial region. "The unique patterns of both surface and subsurface responses reinforce each other, resulting in a notably strong equatorial cooling that persists decades after the removal of anthropogenic aerosols". For how long?

8. Conclusion

Climate science is a science of qualitative approximations, of probable or fuzzy rather than exact results. This fact is shown in the emergence and dynamics of turbulence movements in climate phenomena. An intensification of turbulent phenomena could have important consequences for human activities (weather- heatwaves and floods forecasting, waterworks, restoring natural areas, aviation, urban management, etc.). We attempt here to draw some connections between the theoretical turbulence modeling and the experimental results interpreted using such models and the reality of large-scale natural events strongly related to anthropogenic climate changes, such as heatwaves and the cooling effect of aerosols. In fact we believe that more sophisticated practical results could be drawn from connecting theoretical turbulence studies to natural real phenomena, especially those under the influence of climate change. The most important property of turbulence is its ability to produce mixing, thereby transporting scalar quantities such as heat or aerosols both along and across surfaces of constant density, in the atmosphere or in the ocean. Associated with this fact is that turbulence is a strong interaction phenomenon, and highly dissipative. A striking feature of chaotic dissipative systems such as turbulence is the spontaneous emergence of coherent structures, within the space of chaotic interactions. Understanding the mechanisms of turbulence, dispersion and heat transfer is becoming increasingly important. Although many *in situ* wind and mass concentration measurements are available their utility is limited because specifically turbulence data which should be used in k-ε eddy viscosity model, for instance, are more often than not insufficient and non-homogeneous. Curtailing this fact requires an adequate understanding of the processes that control velocity and turbulence within the surface roughness layer. Indeed, it has to be stressed that in this case as in many other similar ones the measurements of wind velocity are not measurements of turbulent fluctuations, which remain difficult to detect due to their small size and high variability. Vorticity is another related feature. We suggest the question whether the vorticity in the free atmosphere is related to the eddies spectrum? If so, is it a continuum one or does it include some qualitative change. This question might be essential in order to understand the chaotic changes in the atmospheric circulation triggered by turbulence. To date, the large-scale spatio-temporal characteristics of turbulence has yet to be fully understood, due to the lack of sufficient *in situ* detection instruments in the atmosphere. We advance here another question: in which way the basic randomness of turbulence is related to the fact that in many processes, random individual events can, in aggregate, lead to highly deterministic outcomes as indicated by Michel Talagrand? Using Talagrand observation we need to calculate the effects of turbulence on various processes in the atmosphere that may (or may not) interfere significantly with the free circulation flow and find if, when and how the *concentration of measure* pattern exists in the atmospheric turbulence and jet streams. The fragmentary understanding of the physical drivers contributing to heatwaves is aggravated by a fundamental lack of understanding of dynamical aspects. Among the influential factors is the atmospheric circulation, typically considered a fast driver. Studying extreme heat waves then amounts to studying the nonlinear and turbulent dynamics of the atmosphere. Two key dynamical variables to be studied are the temperature and pressure fields. Our climate system is not just a static thermodynamic system, it is a fluid dynamical system. One, if not the major problem resides precisely in the far from complete understanding of the fundamental processes. We are permanently confronted with the need to model different uncertainties using various concepts and methods. All these uncertainties continue compromising the gathering of useful information without which no strategy is reliable. Aerosol–cloud–precipitation interactions represent one of the major uncertainties in weather and climate prediction. The aerosol droplets have acute complex effects. These complexities mean there is still a great deal of uncertainty about the magnitude of the overall cooling from pollutants such as SO_2. But if aerosol cooling is larger than generally assumed, the planet will warm more rapidly than predicted as soon as aerosol levels fall. Understanding the spatial distribution of finite-size massive impurities, such as droplets, dust, or bubbles suspended in incompressible flows is a crucial issue in cloud physics. Let's imagine a stable circulation generated by temperature and/or pressure gradients. The flow at the microscale level hitting the aerosol droplets generates very small eddies which form tiny drag zones. The question we suggest is if these eddies, present in such a big number in the free flow, might interact and form a large pattern of vorticity sufficient to "derail" the stable atmospheric circulation. Many climate models fail to simulate the Eastern Tropical Pacific (ETP) sea surface temperature (SST) wedge-shaped cooling phenomenon. Patterns of both surface and subsurface responses reinforce each other, resulting in a notably strong equatorial cooling that persists decades after the removal of anthropogenic aerosols. For how long?

Abbreviations

MOST: Monin-Obukhov Similarity Theory
GHG: Greenhouse Gas
GCM: Global Climate Models
CCN: Cloud Condensation Nuclei
ETP: Eastern Tropical Pacific
SST: Sea Surface Temperature

Author Contributions

Petre Roman is the sole author. The author read and approved the final manuscript

Conflicts of Interest

The authors declare no conflicts of interest.

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